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Dephasing of Andreev pairs entering a charge density wave

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Abstract. A Cooper pair from a s-wave superconductor (S) entering a conventional charge density wave (CDW) below the Peierls gap dephases on the Fermi wavelength while one particle states are localized on the CDW coherence length ξ_{CDW}. It is thus practically impossible to observe a Josephson current through a CDW. The paths following different sequences of impurities interfere destructively, due to the different electron and hole densities in the CDW. The same conclusion holds for averaging over the conduction channels in the ballistic system. We apply two microscopic approaches to this phenomenon: (i) a Blonder, Tinkham, Klapwijk (BTK) approach for a single highly transparent S-CDW interface; and (ii) the Hamiltonian approach for the Josephson effect in a clean CDW and a CDW with non magnetic disorder. The Josephson effect through a spin density wave (SDW) is limited by the coherence length ξ_{SDW}, not by the Fermi wave-length. A Josephson current through a SDW might be observed in a structure with contacts on a SDW separated by a distance ξ_{SDW}.

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1 Introduction

Condensed matter provides many phases with an energy gap between the ground state and the lowest excited state, and an exponential decay of one-particle correlations. Well-known examples of gapped (super)conducting or insulating phases are superconductivity, the quantum Hall effect, the Haldane gap in quasi-one dimensional (quasi-1D) spin 1 antiferromagnets, charge and spin density waves. The coexistence between different orderings is usually difficult in bulk systems, but the progress in nanofabrication technology allows electron transport experiments in submicron hybrids made of several electrodes with different order parameters. Of particular interest are typical mesoscopic experiments with charge density waves (CDWs) [1–4], such as transport through constrictions [5], through nanowires [6], through an array of holes [7], through normal metal-charge density wave (N-CDW) point contacts [8–10], an Aharonov-Bohm effect experiment [11], and a scanning tunneling microscope experiment [12].

Charge is transported below the superconducting gap by Andreev reflection at a normal metal-superconductor (N-S) interface [13]: a spin-up electron from the normal side is reflected as a hole in the spin-down band and a Cooper pair is transmitted in the superconductor [13]. Andreev pair [14] transport through a 1D metallic channel was realized recently in the form of the Josephson effect through a carbon nanotube [15].

The tunneling current through an insulator decays on the coherence length $\xi = \hbar v_F/\Delta$ (with v_F the Fermi velocity and Δ the charge gap), much larger than the Fermi wavelength λ_F. The dc-Josephson effect through a 1D channel with translational symmetry breaking (a CDW) follows conventional tunneling according to the first approach to this problem by Visscher and Rejaei [1]. Coherent Andreev pair propagation can even be mediated by the sliding motion of the CDW [1,16], suggesting that a mesoscopic CDW can be depinned by a supercurrent. On the other hand, Bobkova and Barash [2] found recently an absence of Andreev bound states at S-CDW-S interfaces. We develop here a microscopic description of Andreev transport in S-CDW hybrids based on the Hamiltonian approach, successfully applied in the recent years to superconducting structures such as for instance a superconducting point contact [17], ferromagnet-superconductor hybrids [18], and to non local transport through a superconductor [19–21]. Single particle evanescent states are localized within the CDW coherence length $\xi_{CDW} = \hbar v_F/|\Delta_{CDW}|$ (with $|\Delta_{CDW}|$ the Peierls gap), much larger than λ_F. Andreev pairs are on the contrary found to dephase on λ_F in a CDW, compatible with reference [2] and not captured by the quasiclassical theory in reference [1]. This conclusion is also obtained from a Blonder, Thinkham, Klapwijk (BTK) approach [3,22].

The dephasing of a pair state in a CDW is obtained in the same framework of the Hamiltonian approach as

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non local Andreev reflection [19–21,23,24] through a superconductor in N-S-N structures [24], a problem relevant to the realization [23,24] of a source of correlated pairs of electrons. The two problems are indeed dual: the former is related to the propagation of an electron pair through a CDW with electron-hole pairing, and the later is related to the propagation in the electron-electron and electron-hole channels through a superconductor with electron-electron pairing. The mechanism for non local transport through a superconductor consists however of *opposite currents* in the electron-electron and electron-hole channels because of the opposite sign of the charge carriers. By contrast, the effect in a CDW is an equilibrium property, that we identify to the dephasing on λ_F of the evanescent pair state. The resulting absence of Josephson effect through a CDW is robust, independent of the interface transparencies, as opposed to being restricted to tunnel interfaces in the superconducting case [21].

The article is organized as follows. A simple physical interpretation of the effect is presented in Section 2. The microscopic model is presented in Section 3. The BTK approach is presented in Section 4. Boundary conditions at interrupted chains and the supercurrent are discussed in Section 5. Concluding remarks are given in Section 6.

2 Physical picture

Let us first consider non magnetic impurities in a CDW. For the sake of simplification, the impurity potential is supposed to be weak enough for the CDW phase to be the same as in the absence of impurities. The discussion of localized phase deformations due to strong pinning impurities is given in Section 5.2.5. Phase coherent Andreev reflection at normal metal-superconductor interfaces implies that the backscattered hole in the normal metal follows the same configure of impurities as the incoming electron, in such a way as the different paths followed by an Andreev pair do not dephase with each other, except for finite energy effects controlled by the Thouless energy, and for inelastic scattering.

By contrast, we show that in a CDW, the random phase factors acquired by a spin-up electron visiting different impurities do not cancel with the phase of a spin-down hole visiting the same sequence of impurities, leading to dephasing of the Andreev pair. The microscopic model discussed below in the ballistic system shows that dephasing occurs on the smallest length scale: the Fermi wave length, (up to a factor of two) equal to the period of the CDW modulation. The dephasing of an Andreev pair has its origin in the fact that the total number of spin-up electrons at position x along the chain, given by $N_{e,\uparrow}^{(CDW)}(x) = N_0 + N_1 \cos(Qx+\varphi)$ is dephased by π compared to the the total number of spin-down holes $N_{h,\downarrow}^{(CDW)}(x) = N_0' + N_1 \cos(Qx+\varphi+\pi) = N_0' - N_1 \cos(Qx+\varphi)$, because a maximum in the number of electrons corresponds to a minimum in the number of holes. The quantities N_0, N_0' and N_1 are respectively the number of normal electrons and holes, and the amplitude of the modulation in the number of electrons. The total number of available states is $N_0 + N_0'$.

Let us consider a single non magnetic impurity in a CDW. The impurity contribution to the energy of a spin-up electron is

$$\begin{aligned}\mathcal{H}_{e,\uparrow}^{(imp)} &= V(x_i)\left[N_{e,\uparrow}^{(CDW)}(x_i) - N_0\right] \\ &= V(x_i) N_1 \cos(Qx_i+\varphi)\end{aligned} \tag{1}$$

equal to the same quantity for a spin-down hole:

$$\begin{aligned}\mathcal{H}_{h,\downarrow}^{(imp)} &= -V(x_i)\left[N_{h,\downarrow}^{(CDW)}(x_i) - N_0'\right] \\ &= V(x_i) N_1 \cos(Qx_i+\varphi),\end{aligned} \tag{2}$$

where x_i is the position of the impurity and $V(x_i)$ the disorder impurity potential. The N_0 and N_0' terms that were subtracted the normal ordered equations (1) and (2) induce an exactly opposite dephasing for an electron and a hole following the same sequence of impurities, so these terms do not dephase the Andreev pair at equilibrium. The N_1 terms are on the contrary additive for electrons and holes making an Andreev pair (see Eqs. (1) and (2)), resulting in a dephasing between the different paths following different sequences of impurities. As we show below by explicit calculations, the same conclusion holds for a ballistic multichannel system, where the phase factors have their origin in Friedel oscillations.

A spin density wave (SDW) can be described as two out-of-phase CDWs for spin-up and spin-down electrons. The number of spin-up electrons is $N_{e,\uparrow}^{(SDW)}(x) = N_0 + N_1 \cos(Qx+\varphi)$, and the number of spin-down electrons is $N_{e,\downarrow}^{(SDW)}(x) = N_0 - N_1 \cos(Qx+\varphi)$. The total density is not modulated, but the spin density is modulated. The number of spin-down holes $N_{h,\downarrow}^{(SDW)}(x) = N_0 + N_1 \cos(Qx+\varphi)$ is equal to $N_{e,\uparrow}^{(SDW)}(x)$, the number of spin-up electrons. We conclude by the preceding argument that non magnetic impurity random phases of spin-up electrons and spin-down holes cancel with each other in the total phase of the Andreev pair propagating through a SDW, so that a Josephson effect over the coherence length is possible in a SDW.

We provide now three different microscopic approaches to the absence of Andreev pair transport through a ballistic CDW. Disorder is treated in Appendix B.

3 The model

The microscopic theory is based on the electronic part of the 1D Peierls Hamiltonian of a ballistic CDW:

$$\begin{aligned}\mathcal{H} = &-\sum_{n,\sigma}\left(t_0 + |\Delta_{CDW}|\cos(2k_F x_n)\right) \\ &\times\left(c_{n+1,\sigma}^{+} c_{n,\sigma} + c_{n,\sigma}^{+} c_{n+1,\sigma}\right) - \mu \sum_{n,\sigma} c_{n,\sigma}^{+} c_{n,\sigma},\end{aligned} \tag{3}$$

where t_0 is the mean hopping amplitude, k_F the Fermi wave-vector, and μ the chemical potential. The summation over the integer n runs over the sites of the 1D chain. We have $x_n = na_0$, with a_0 the lattice parameter in the absence of the CDW modulation. We suppose an incommensurate charge density wave, unless specified otherwise in the discussion of edge states.

4 Blonder, Tinkham, Klapwijk approach

We evaluate now within the BTK approach [22] subgap transport at a S-CDW interface, which was already probed experimentally in reference [47]. A BTK approach to N-CDW interfaces can be found in reference [3]. A scattering approach to S-CDW interfaces with unconventional superconductors and charge density waves can be found in reference [2]. To describe the CDW and superconducting correlations on an equal footing, we introduce a four component wave-function corresponding to the four creation and annihilation operators $c^+_{k,R,\uparrow}$, $c^+_{k,L,\uparrow}$, $c_{k,R,\downarrow}$ and $c_{k,L,\downarrow}$ of right (R) and left (L) moving spin-σ electrons ($\sigma = \uparrow, \downarrow$) of wave-vector k. The wave-function in the CDW part of the junction is given by

$$\psi_{CDW}(x<0) = \begin{pmatrix} ue^{i\varphi} \\ v \\ 0 \\ 0 \end{pmatrix} e^{\left(ik_F + \frac{1}{\xi_{CDW}}\right)x} \quad (4)$$

$$+b \begin{pmatrix} ue^{i\varphi} \\ v \\ 0 \\ 0 \end{pmatrix} e^{-\left(ik_F - \frac{1}{\xi_{CDW}}\right)x} + b' \begin{pmatrix} v^* \\ u^* e^{-i\varphi} \\ 0 \\ 0 \end{pmatrix} e^{\left(ik_F + \frac{1}{\xi_{CDW}}\right)x}$$

$$+a \begin{pmatrix} 0 \\ 0 \\ ue^{i\varphi} \\ v \end{pmatrix} e^{\left(ik_F + \frac{1}{\xi_{CDW}}\right)x} + a' \begin{pmatrix} 0 \\ 0 \\ v^* \\ u^* e^{-i\varphi} \end{pmatrix} e^{\left(-ik_F + \frac{1}{\xi_{CDW}}\right)x},$$

where x is the coordinate along the chain, u and v are the CDW coherence factors, and u^* and v^* their complex conjugate. The wave-functions in the superconductor take the form

$$\psi_S(x>0) = d \begin{pmatrix} u_0 \\ 0 \\ v_0 \\ 0 \end{pmatrix} e^{(ik_F - \frac{1}{\xi_s})x}$$

$$+ d' \begin{pmatrix} 0 \\ u_0 \\ 0 \\ v_0 \end{pmatrix} e^{-(ik_F + \frac{1}{\xi_s})x} + c \begin{pmatrix} v_0 \\ 0 \\ u_0 \\ 0 \end{pmatrix} e^{-(ik_F + \frac{1}{\xi_s})x}$$

$$+ c' \begin{pmatrix} 0 \\ v_0 \\ 0 \\ u_0 \end{pmatrix} e^{(ik_F - \frac{1}{\xi_s})x},$$

where u_0 and v_0 are the BCS coherence factors. Matching the wave-functions and their derivatives for highly transparent interfaces leads to $c = d = c' = d' = a' = a = b = 0$

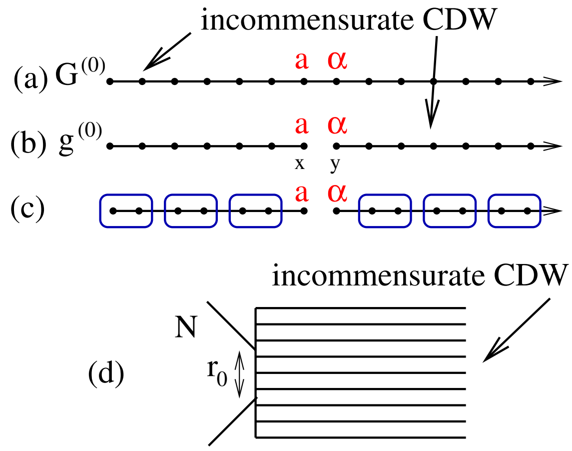

Fig. 1. Schematic representations of (a): a fully connected CDW chain; (b): a disconnected chain; (c): a disconnected dimerized state with zero energy edge state levels at sites a and α and a dimer order parameter along the chain [25,26]; (d): a junction of cross section area $\sim r_0^2$ between a normal metal and an incommensurate CDW with the chains perpendicular to the interface. See reference [10] for the experimental realization of (d). For clarity, (c) is drawn for a dimerization, but the model applies to the more general case of incommensurate modulations.

and $|b'|^2 = 1$. No charge is transported by the reflection of a right-moving CDW quasiparticle in a left-moving quasiparticle, as for a N-CDW interface [8,9].

A pair from the superconductor decomposes on pairs of evanescent CDW quasiparticles. The resulting forward and backward combinations are both spin singlets but interfere destructively in the CDW, in such a way as to produce an absence of Andreev pair penetration in a CDW.

5 Hamiltonian approach

5.1 Edge states

We discuss now the boundary conditions at the extremity of a finite chain before considering the supercurrent in Section 5.2. A CDW of finite length is obtained by disconnecting an infinite chain as in Figures 1a and 1b [18]. The Green's functions of an infinite CDW are evaluated in Appendix A. The advanced Green's functions of the connected (disconnected) chain are denoted by $G^{A,e,\uparrow}(\omega)$ ($g^{A,e,\uparrow}(\omega)$) (see Fig. 1a). They are related to each other by the Dyson equation

$$G^{A,e,\uparrow}_{a,\alpha}(\omega) = g^{A,e,\uparrow}_{a,\alpha}(\omega) + g^{A,e,\uparrow}_{a,a}(\omega) t_{a,\alpha} G^{A,e,\uparrow}_{\alpha,\alpha}(\omega). \quad (5)$$

The condition that the chain in Figure 1b is disconnected is expressed by $g^{A,e,\uparrow}_{a,\alpha}(\omega) = 0$. We use the notations $t_{a,\alpha} = t_0 + |\Delta_{CDW}| \cos(2k_F x)$, $G^{A,e,\uparrow}_{a,\alpha}(\omega) = g^{A,e,\uparrow}_{x,y}(\omega)$, and $G^{A,e,\uparrow}_{\alpha,\alpha}(\omega) = g^{A,e,\uparrow}_{y,y}(\omega)$, $g^{A,e,\uparrow}_{x,y}(\omega)$ is defined by equation (A.1), and the neighboring sites a and α are at coordinates x and y (see Figs. 1a and 1b). We deduce $g_{a,a}^{A,e,\uparrow}(\omega) = G_{a,\alpha}^{A,e,\uparrow}(\omega)/[t_{a,\alpha} G_{\alpha,\alpha}^{A,e,\uparrow}(\omega)]$, leading to

$$g_{a,a}^{A,e,\uparrow}(\omega) \simeq \frac{\sin(k_F a_0)\sqrt{|\Delta_{CDW}|^2 - \omega^2}}{t_{a,\alpha}(\omega - \omega_0 - i\eta)} \tag{6}$$

for $\omega \simeq \omega_0$, where η is small and positive. We obtain a state of energy $\omega_0 = |\Delta_{CDW}| \cos(\varphi_{x,y})$, with $\varphi_{x,y} = \varphi + k_F(x+y)$, localized in a region of size ξ_{CDW} at the extremity of the chain.

We recover known results for a dimerized chain. In the limit of a strong dimerization, a semi-infinite chain ends either as a dimer or as an isolated site (as in Fig. 1c), resulting for the later in an edge state at zero energy at the extremity of each semi-infinite chain, corresponding to $\varphi_{x,y} = \pi/2$ and $\omega_0 = 0$ in equation (6). This shows the consistency between the Hamiltonian approach [17,18] used here for CDW hybrids, and the known behavior of a dimerized system [25,26].

Considering now the incommensurate case, the phase $\varphi_{x,y}$ entering the expression of ω_0 in equation (6) is treated as a random variable because of disorder in the position of the extremities of the chains at a multichannel N-CDW contact (see Fig. 1d). A uniform distribution of $\varphi_{x,y}$ leads to a uniform subgap density of edge states at site a (see Fig. 1): $\rho_{a,a}(\omega) = \sin(k_F a_0)/\pi t_0$, as compared to $\rho_{a,a}^N(\omega) = 1/\pi t_0$ in the normal state. The CDW pair amplitude at the extremity of the semi-infinite chain is

$$F_{a,a}(|\Delta_{CDW}|, \omega) = \frac{|\Delta_{CDW}| \sin(k_F a_0)}{t_{a,\alpha}^{(0)} \left[(\omega - i\eta)^2 - |\Delta_{CDW}|^2 \cos^2(\varphi_{x,y})\right]} \times \left\{\omega \sin(\varphi_{x,y}) - \sqrt{|\Delta_{CDW}|^2 - \omega^2} \cos(\varphi_{x,y})\right\}. \tag{7}$$

The pair amplitude integrated over energy, and therefore the self-consistent Peierls gap, vanish in a given window of the phase $\varphi_{x,y}$, leading to normal states at the CDW boundary because of interrupted chains. The resulting subgap conductance at a N-CDW interface in the geometry in Figure 1d is not contradicted by available experiments on N-CDW contacts [10]. Normal states at the extremity of an interrupted chain are also compatible with the appearance of a normal region around a nanohole induced by columnar defects in a CDW film, which was proposed [3] to explain the Aharonov-Bohm oscillation experiment in a CDW [11].

5.2 Supercurrent

Now we evaluate the transport of Andreev pairs through a CDW in the form of the Josephson effect in the device in Figure 2. The superconductors are described by BCS theory, with a superconducting gap much smaller than the CDW gap, as in possible experiments. The dc-supercurrent per conduction channel for arbitrary interface transparencies is given by

$$I_S = \frac{e}{2\hbar} \int_0^{+\infty} \frac{d\omega}{2i\pi} \mathrm{Im}\left\{\mathrm{Tr}\left\{\hat{\sigma}_3 \left[\hat{t}_{a,\alpha} \hat{G}_{\alpha,\alpha}^A, \hat{t}_{\alpha,a} \hat{g}_{a,a}^A\right]\right\}\right\} \tag{8}$$

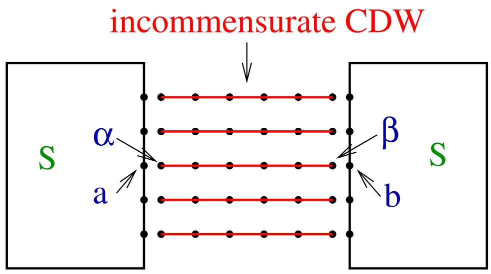

Fig. 2. Schematic representation of a S-CDW-S junction between a superconductor (S), a charge density wave (CDW) and another superconductor (S).

where $[\,,]$ is a commutator, $\hat{\sigma}_3$ is one of the Pauli matrices in Nambu space, and the trace is a sum over the four components of the matrix Green's functions, corresponding to spin degenerated right-left and electron-hole degrees of freedom.

5.2.1 Tunnel interfaces

We first consider lateral atomic contacts connecting two superconductors to an infinite CDW. The CDW lattice is supposed to be weakly modified by the contacts, with no edge state. The dc-supercurrent in the tunnel limit is $I_S = I_{CDW} \sin\varphi$, with φ the superconducting phase difference, and with the critical current

$$I_{CDW} = \frac{e}{h} \Delta_S T^2 t_0^2 \overline{g_{\alpha,\beta}^{A,e,\uparrow}(\Delta_S) g_{\beta,\alpha}^{A,h,\downarrow}(\Delta_S)}, \tag{9}$$

where T is the normal state transparency (with both the CDW and the superconductor in the normal state), $g_{\alpha,\beta}^{A,e,\uparrow}(\Delta_S)$ (see Eq. (A.1)) and $g_{\beta,\alpha}^{A,h,\downarrow}(\Delta_S)$ are the advanced spin-up electron and spin-down hole CDW Green's functions at energy $\omega = \Delta_S$, where Δ_S is the superconducting gap (α and β are shown in Figure 2, and t_0 is defined by equation (3)). Averaging over the Fermi oscillations due to the large number of conduction channels in parallel in the incommensurate case is denoted by an overline in equation (9). We deduce from equation (A.1), and from a similar expression for $g_{\beta,\alpha}^{A,h,\downarrow}(\Delta_S)$ that the Andreev pair propagator $\overline{g_{\alpha,\beta}^{A,e,\uparrow} g_{\beta,\alpha}^{A,h,\downarrow}}$, and hence the tunnel supercurrent vanish as soon as the distance between the superconducting electrodes exceeds λ_F (see Appendix A), in agreement with the preceding sections.

5.2.2 Arbitrary interface transparencies

To describe arbitrary interface transparencies, we treat multiple scattering at each interface to all orders [21] while neglecting multiple Andreev reflections [17] because of the damping of the propagation in the CDW. Within this approximation, the dressed 4×4 Green's function $\hat{\mathcal{G}}_{\alpha,\beta}$ is $\hat{\mathcal{G}}_{\alpha,\beta} = \hat{\mathcal{M}}_\alpha^{-1} \hat{g}_{\alpha,\beta} \hat{\mathcal{N}}_\beta^{-1}$, with $\hat{\mathcal{M}}_\alpha = \hat{I} - \hat{g}_{\alpha,\alpha} \hat{t}_{\alpha,a} \hat{g}_{a,a} \hat{t}_{a,\alpha}$ and

$\hat{\mathcal{N}}_\beta = \hat{I} - \hat{t}_{\beta,b}\hat{g}_{b,b}\hat{t}_{b,\beta}\hat{g}_{\beta,\beta}$, where $\hat{t}_{a,\alpha}$ $[\hat{t}_{b,\beta}]$ are the diagonal hopping amplitude matrices with elements $t_{a,\alpha}$ $(t_{b,\beta})$ for electrons, and $-t_{a,\alpha}$ $(-t_{b,\beta})$ for holes. We deduce from equation (9) that the supercurrent vanishes beyond λ_F whatever the interface transparencies.

5.2.3 Sliding motion

Let us suppose now that a finite voltage is applied between the two superconductors connected by low transparency interfaces to a sliding CDW [1]. The ac-Josephson effect is treated by a gauge transformation in which the time-dependent superconducting and CDW phases $\varphi(t)$ and $\chi(t)$ at time t are absorbed in the hopping matrix elements [1,17]. The supercurrent, obtained from the Keldysh Green's function, is expanded as in the S-N-S case in terms of the harmonics $G_{r,s}(\omega) = \tilde{G}(\omega + r\omega_0/2, \omega + s\omega_0/2)$ of the Fourier transform of the Green's function $G(t,t')$, with $\chi(t) - \varphi(t) = \omega_0 t$, where we supposed $\chi(t)$ linear in t. The supercurrent contains the gauge transformed Andreev pair propagator in the CDW, limited by λ_F as in the dc-case.

5.2.4 Edge states

A finite length in the CDW chain (see Fig. 2) also does not restore longer range Andreev pair transport. The Green's functions $g_{\alpha,\beta}$ and $g_{\beta,\alpha}$ of the finite chain are indeed related to the Green's functions $G_{a,\beta}$ and $G_{\beta,a}$ of the infinite chain by $g_{\alpha,\beta} = G_{a,\beta}/[G_{a,a}t_{a,\alpha}]$ and $g_{\beta,\alpha} = G_{\beta,a}/[G_{a,a}t_{\alpha,a}]$ (see the notations in Fig. 2). The supercurrent of the finite chain is proportional to the Andreev pair propagator of the infinite chain, again limited by λ_F. Normal states of range ξ_{CDW} were obtained at both interfaces if the CDW gap is self-consistent. The pair current does however not propagate through the gapped region between the edges. This contrasts with the Josephson effect through a discrete state in a nanotube of finite length [15].

5.2.5 Bulk defects

A supercurrent may propagate through a perturbed region of size ξ_{CDW} around the defects [27–29], but does not propagate through the regions with no defect. Hopping between possible normal regions in the CDW [29] does not restore a longer range Josephson effect.

5.2.6 Weak disorder in the Born approximation

Disorder treated in the Born approximation [30] is relevant to the description of the effect of disorder on the sliding motion. As shown in Appendix B, the supercurrent vanishes also for disorder in the Born approximation.

6 Discussion and conclusion

To conclude, we have shown that pair states are dephased within λ_F in a CDW, in spite of one particle states localized on ξ_{CDW}. The absence of Josephson effect through a CDW is an intrinsic property of a ballistic multichannel multichannel or disordered single channel CDW, robust against highly transparent interfaces, finite voltages, and disorder. The supercurrent through a single channel CDW shows oscillations on a length scale $\sim\lambda_F$. Hopping between normal states due to a self-consistent CDW gap vanishing locally because of edges or bulk defects, does not allow to overcome the absence of pair propagation in the gapped regions. Transport of Andreev pairs is possible through a SDW.

Spin active interfaces induce a Josephson current through a half-metal ferromagnet [31,32] because of the propagation of superconducting correlations among pairs of electrons with the same spin orientation [33–35]. Spin active S-CDW interfaces, or magnetic scattering in the CDW (see Refs. [36,37] for blue bronzes), would change the spin-down Andreev reflected quasi-hole in a spin-up quasi-hole while preserving charge, therefore preserving the absence of Josephson current.

The absence of Andreev pair penetration in a charge density wave that we discussed is compatible with the fact that, in bulk systems, superconductivity hardly coexists with charge density waves on the same portion of the Fermi surface for small disorder. Other mechanisms are likely to be responsible for the experimentally observed coexistence between superconductors and CDWs in a number of compounds [38]. For instance, besides a possible coexistence between superconductivity and charge density wave in layered compounds and similar structures [39,40], a bulk coexistence [41] is possible in the presence of a sufficient non magnetic disorder [42]. Andreev pair transport through a CDW is also possible if the concentration of non magnetic impurities in the CDW is such that the CDW gap becomes smaller than the superconducting gap (see Appendix B).

A parallel can be drawn with ferromagnet-superconductor (FS) structures. In mesoscopic structures, the Josephson current through a ferromagnet shows oscillations leading to π-junctions [43], and, in bulk structures, the coexistence between superconductivity and ferromagnetism can lead to a Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) state [44,45] with a spatially modulated gap. On the other hand, it was proposed that, in addition to a uniform superconducting gap, a FFLO [44,45]-like superconducting gap modulated at the CDW wave-vector Q could coexist with the CDW. The spatial oscillations of the Josephson current obtained in Section 5 in mesoscopic S-CDW-S structures can be viewed as the CDW counterpart of the oscillations of the Josephson current in S-F-S structures [43] that average to zero in the case of strong ferromagnets because of their very short period.

Regarding the possibility of transporting Andreev pairs through a spin density wave that we obtained here, the compatibility between s-wave superconductivity and SDWs can be seen in bulk systems from the transition between a SDW and a superconductor in the phase diagram of the series of compounds $(TMTTF)_2X$ and $(TMTTSF)_2X$ under pressure [46].

From the point of view of experiments, the model of edge states discussed above is a possibility for interpreting the conductance spectra of N-CDW point contacts [10]. The realization of a point contact between a superconductor and a CDW is possible with the same technology as in reference [10] for a N-CDW point contact, but with lower temperatures. We expect a normal current as in reference [10] if the temperature is such that the superconductor is in the normal state or if the applied voltage is larger than the superconducting gap. From our model, no coherent transport of Andreev pairs is possible in the CDW if the voltage is within the superconducting gap. This is compatible with the sharp increase of differential resistance within the superconducting gap observed experimentally in Figures 1c–1e of reference [47] for highly transparent Nb-$NbSe_3$ point contacts. The realization of Josephson junctions with CDWs or SDWs, more technically involved, requires a very short distance between the superconducting electrodes. Finally, we note that, interestingly, two interacting particles on a one dimensional quasi-periodic lattice lead to two-particle localized states with quasi-delocalized one particle states [48].

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Appendix A: Green's functions of a charge density wave

The elements of the 2×2 advanced Green's function [30] of a spin-up electron, connecting two lattice sites at positions x and y at energy ω with respect to the chemical potential μ are denoted by $g_{x,y}^{A,e,\uparrow,(i,j)}(\omega)$, with $i, j = R, L$, corresponding to right and left moving fermions respectively. Evaluating the total advanced Green's function obtained by summing over i and j leads to

$$g_{x,y}^{A,e,\uparrow}(\omega) = \frac{1}{2t_0} e^{-R/\xi_{CDW}(\omega)} \left[-\frac{\omega}{s(\omega)} \cos(k_F R) - \sin(k_F R) + \frac{|\Delta_{CDW}|}{s(\omega)} \cos(\varphi + k_F(x+y)) \right], \quad (A.1)$$

with $R = x - y$, $\Delta_{CDW} = |\Delta_{CDW}| \exp(i\varphi)$ the complex CDW order parameter, and $s(\omega) = \sqrt{|\Delta_{CDW}|^2 - \omega^2}$. The absence of Josephson effect discussed in Section 5.2.1 for tunnel interfaces is obtained by noting that the Green's function of a spin-down hole is

$$g_{x,y}^{A,h,\downarrow}(\omega) = \frac{1}{2t_0} e^{-R/\xi_{CDW}(\omega)} \left[-\frac{\omega}{s(\omega)} \cos(k_F R) - \sin(k_F R) - \frac{|\Delta_{CDW}|}{s(\omega)} \cos(\varphi + k_F(x+y)) \right], \quad (A.2)$$

where the hole CDW phase been has changed [49] by π compared to the electron CDW phase in equation (A.1) (see Sect. 2).

Averaging over the conduction channels leads to

$$\overline{g_{x,y}^{A,e,\uparrow}(\omega) g_{y,x}^{A,h,\downarrow}(\omega)} = 0 \quad (A.3)$$

for $|x - y|$ exceeding λ_F.

By contrast, in the SDW case, the Andreev pair propagator reduces to $\overline{[g_{x,y}^{A,e,\uparrow}(\omega)]^2}$ because the phase of spin-down electrons is shifted by π compared to the CDW case. We find easily

$$\overline{\left[g_{x,y}^{A,e,\uparrow}(\omega) \right]^2} = \frac{1}{4t_0^2} \frac{|\Delta_{CDW}|^2}{|\Delta_{CDW}|^2 - \omega^2} \exp\left(-\frac{2R}{\xi_s(\omega)} \right), \quad (A.4)$$

where $\xi_s(\omega)$ is the SDW coherence length. The supercurrent through a SDW therefore decays over ξ_s, not over λ_F as for a CDW, in agreement with Section 2.

Appendix B: Disorder in the Born approximation

The 2×2 Green's functions for the right-left components of a spin-up electron are given by $\hat{G} = \hat{g} + \hat{g}\hat{\Sigma}\hat{G}$ in the Born approximation, with the self-energy $\hat{\Sigma} = \int (dk/2\pi) \hat{v} \hat{g}(k,\omega) \hat{v}^+$, where $\hat{g}(k,\omega)$ is the ballistic matrix Green's function and $\hat{v}$ the matrix containing the forward and backward scattering amplitudes. Following reference [30], we find

$$\hat{\Sigma} = \int \frac{dk}{2\pi} \left[|u|^2 \begin{pmatrix} g_{R,R}(k,\omega) & g_{R,L}(k,\omega) \\ g_{L,R}(k,\omega) & g_{L,L}(k,\omega) \end{pmatrix} + |v|^2 \begin{pmatrix} g_{L,L}(k,\omega) & 0 \\ 0 & g_{R,R}(k,\omega) \end{pmatrix} \right], \quad (B.1)$$

with u and v the amplitudes of backward and forward scattering. We deduce the Green's function of a spin-up electron:

$$G^{A,e,\uparrow}(\xi_k, \omega) = \frac{\overline{\omega} + \overline{\xi}_k \hat{\tau}_3 + \overline{\Delta_{CDW}} \hat{\tau}^+ + \overline{\Delta_{CDW}}^* \hat{\tau}^-}{\overline{\omega}^2 - |\overline{\Delta_{CDW}}|^2 - (\overline{\xi}_k)^2}, \quad (B.2)$$

with

$$\overline{\Delta_{CDW}} = \Delta_{CDW} \left[1 - \frac{1}{\tau s(\omega)} \right] \quad (B.3)$$

$$\overline{\omega} = \omega \left[1 + \frac{1}{\tau s(\omega)} + \frac{1}{\tau' s(\omega)} \right] \quad (B.4)$$

$$\overline{\xi}_k = \xi_k + \alpha, \quad (B.5)$$

with $\tau\overline{|u|^2}/\hbar v_F = 1$, $\tau'\overline{|v|^2}/\hbar v_F = 1$, ξ_k the kinetic energy with respect to the chemical potential, α a shift in the chemical potential, and where $\overline{\Delta_{CDW}}^*$ is the complex conjugate of $\overline{\Delta_{CDW}}$. The matrices $\hat{\tau}_3$, $\hat{\tau}^+$ and $\hat{\tau}^-$ are given by

$$\hat{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (B.6)$$

$$\hat{\tau}^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad (B.7)$$

$$\hat{\tau}^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \quad (B.8)$$

The Green's function of a spin-down hole is obtained by changing Δ_{CDW} in $-\Delta_{CDW}$ as in reference [1]. An electron-hole transmission coefficient is evaluated as in reference [50] for a superconductor. Equation (B.2) leads to $\int(dk/2\pi)G^{A,e,\uparrow}(k,\omega)G^{A,h,\downarrow}(k,\omega) = 0$, obtained from evaluating the matrix products and the integral over wave-vector. This identity can be understood from the numerators of the normal and anomalous contributions in $G^{A,e,\uparrow}(\xi_k,\omega)G^{A,h,\downarrow}(\xi_k,\omega)$, with the constraint $\overline{\omega}^2 - |\Delta_{CDW}|^2 - (\overline{\xi}_k)^2 = 0$. We conclude that the Andreev propagator is limited by the elastic mean free path since the transmission coefficient in the ladder approximation vanishes after a single impurity scattering coupling the spin-up electron to the spin-down hole branches.

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